

*Note on a Queueing Model of
Delta Networks*

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This is intended as an addendum to *A Queueing Model of Delta Networks* (SEL Report No. 159), where it was assumed that packets rejected from multistage interconnection networks (ICN's) are sent back to their source for resubmission. This approach "penalizes" packets lost in the latter stages of the network more severely and may be misleading in overestimating actual *PDT* (packet delay time) for ICN's which do exhibit blocking. This addendum describes a modification which may be more realistic in predicting *PDT* when blocking occurs.

As an approximation to the rejection/blocking phenomenon assume that a packet lost in state i ($2 \leq i \leq z$) is sent back to the input of its stage $i - 1$ queue. This should be a feasible approximation because a packet leaving stage $i - 1$ leaves an open position in its queue, and if it is rejected at the input of stage i , it may return to stage $i - 1$ where its position will still be available with high probability. If the rejection is practically instantaneous, then by the approximation of Poisson processes, another packet will arrive at the same time with probability 0 (or close to it). Hence the assumption of a single stage rejection delay seems reasonable.

Revising the Bernoulli resubmission approximation of equation (5) of SEL 159:

$$PDT \approx \sum_{i=1}^z E[T_i] + \sum_{i=2}^z \left(\frac{PL_i}{1 - PL_i} \right) E[T_{i-1}].$$

$\frac{PL_i}{1 - PL_i}$ is the average number of times that a packet must try to enter a queue in stage i before it is accepted. Each rejection takes $E[T_{i-1}]$ time units as its mean time for retry.

Or,

$$\begin{aligned}
 PDT &\approx E[T_z] + \sum_{i=2}^z \left(1 + \frac{p_{Li}}{1 - p_{Li}} \right) E[T_{i-1}] \\
 &= E[T_z] + \sum_{i=2}^z \frac{E[T_{i-1}]}{1 - p_{Li}}.
 \end{aligned}$$

Which leads to:

Case I

$$\begin{aligned}
 E[T_i] &= E[T] \quad i = 1, \dots, z \\
 p_L = p_{Li} &= \frac{1 - \rho}{1 - \rho^{L+1}} \rho^L \quad i = 1, \dots, z
 \end{aligned}$$

So

$$\begin{aligned}
 PDT &= E[T] + \left(\frac{1 - \rho^{L+1}}{1 - \rho^L} \right) (z - 1) E[T] \\
 &= \left[1 + (z - 1) \left(\frac{1 - \rho^{L+1}}{1 - \rho^L} \right) \right] E[T]
 \end{aligned}$$

Case II

$$\begin{aligned}
 PDT &= E[T_2] + \frac{E[T_1]}{1 - p_{L2}} + \sum_{i=3}^z \frac{E[T_2]}{1 - p_{L2}} \\
 &= \frac{E[T_1]}{1 - p_{L2}} + \left(\frac{z - 2}{1 - p_{L2}} + 1 \right) E[T_2]
 \end{aligned}$$

Case III is handled in a similar manner as discussed in SEL 159.

This technique for approximating network blocking seems to be a more reasonable technique than previously presented.

